Pushdown Flow Analysis of First-Class Control

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Flow analysis is instrumental in building good software.

Optimization

Debugging

Verification

Development

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What are currently the options for higher-order flow analysis?

Finite-state models

k-CFA [Shivers 91] and successors.

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Call/return mismatch.

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But in a higher-order language, like Scheme or JavaScript, call/return is the *fundamental* control-flow mechanism.

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Approximate a program as a PDA.
Use the stack for return-point information.
Unbounded call/return matching.

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A pushdown flow analysis [Sharir–Pnueli 81, Reps et al. 95].

First-class functions, tail calls.

Scheme implementation

- More precise than k-CFA
- Usually smaller state space

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Many constructs break call/return nesting:

- Generators (JavaScript, Python)
- Coroutines (Lua, Simula67)
- First-class continuations (Scheme, SML/NJ, Scala)

Finite-state models

✗ Call/return mismatch, many spurious flows

✓ First-class control

Pushdown models

✓ Call/return matching, precise

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Our contribution

✓ Call/return matching, precise
✓ First-class control

Overview

Background on pushdown models

Restricted continuation-passing style (RCPS)

Abstract semantics for RCPS

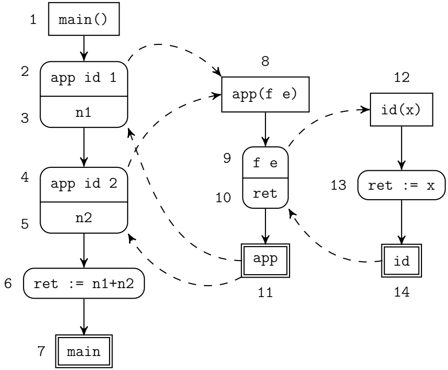
Generalizing summarization

Why pushdown models?

```
(define app (λ (f e) (f e)))
(define id (λ (x) x))

(let* ((n1 (app id 1))
       (n2 (app id 2)))
  (+ n1 n2))
```

Why pushdown models?



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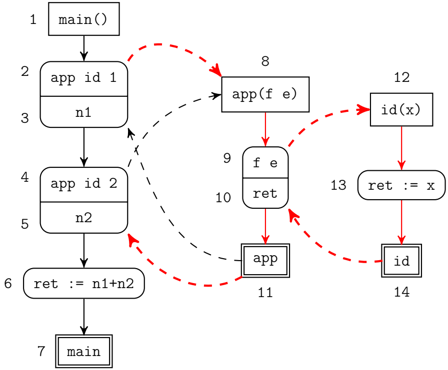

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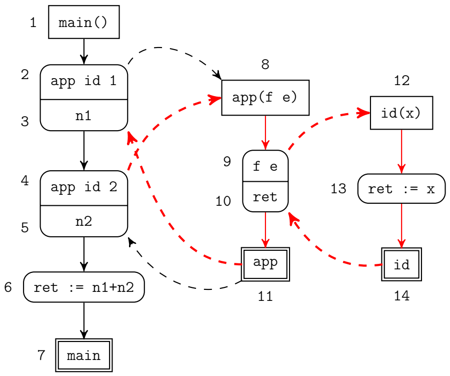

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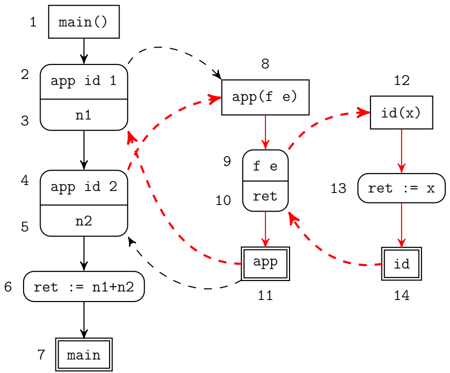

Call/return mismatch causes spurious flow of data
⇒ commonly called functions pollute the analysis.

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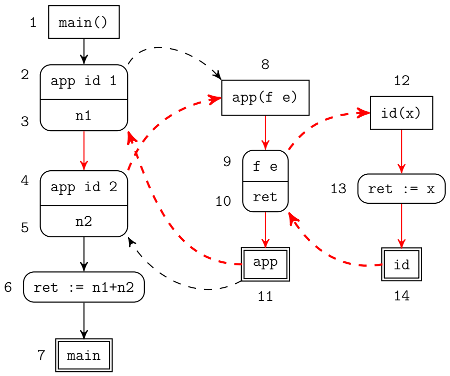

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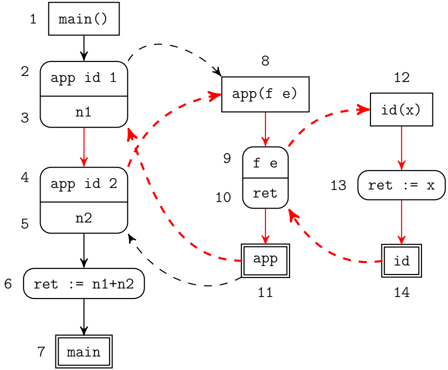

Call/return mismatch causes spurious control flow
$\Rightarrow$ cannot accurately calculate stack change.

Summarization

```
    (define (g x)
        ⋮  )

    (define (f y)
        ⋮
L1:  (g 5 -3)
        ⋮
L2:  (g "a" 2)
        ⋮
L3:  (g 12 7)
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What if g calls an escaped continuation?

The target may not even be on the stack.

Continuation-passing style

Each term is either a user or a continuation term.

```
(define (fact n k)
  (if (= n 0)
      (k 1)
      (fact (- n 1) (λ (ans) (k (* n ans))))))
```

Escaping continuations in CPS

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Manage CPS with a stack [Kranz et al. 86, Orbit].
Stack change from birth to use can be arbitrary.

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Def: a continuation variable can appear free in a user lambda in operator position only.

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Can prove that continuation arguments live on the stack.
Force arbitrary stack change to happen only at continuation calls.

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Abstract interpretation of programs in RCPS λ-calculus.

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Concrete semantics	Actual program behavior
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Abstract semantics	Reminiscent of a PDA, infinite state space
$\Downarrow$	
Local semantics + summarization	No stack, finite state space Weaves calls and returns together

Semantics for escaping continuations

Control enters a user function:

$$(\llbracket (\lambda_l (u\, k)\ call) \rrbracket, \hat{d}, \hat{c}, st, h) \leadsto (call, st', h')$$

$$st' = push([u \mapsto \hat{d}][k \mapsto \hat{c}], st)$$

$$h'(v) = \begin{cases} h(u) \cup \hat{d} & (v = u) \wedge H_?(u) \\ h(k) \cup \{(\hat{c}, st)\} & (v = k) \wedge H_?(k) \\ h(v) & o/w \end{cases}$$

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Calling a continuation:

$$(\llbracket (q\ e)^{\gamma} \rrbracket, st, h) \rightsquigarrow (\hat{c}, \hat{d}, st', h)$$

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Summarization for RCPS

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Traditional summaries: from the entry of λ_2 to `(k1 e)`.

Instead, record entries of λ_1 as we see them.
Create cross-procedure summaries from λ_1 entries to `(k1 e)`.

Summarization for RCPS

```
(λ₁ (x k1) ...(λ₂ (y k2) ...(k1 e) ...)  ...)
```

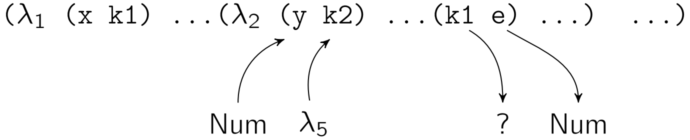
Summarization for RCPS

(λ_1 `(x k1) ...`(λ_2 `(y k2) ...(k1 e) ...)  ...)`

Num $\quad \lambda_5$

Callers: (λ_2, λ_5, Num)
Summaries:

Summarization for RCPS



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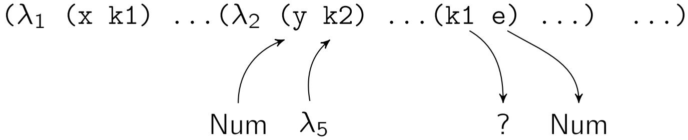

Callers: (λ_2, λ_5, Num), (λ_1, λ_4, Str), (λ_1, λ_7, Bool)
Summaries: (λ_1, Str, Num), (λ_1, Bool, Num)

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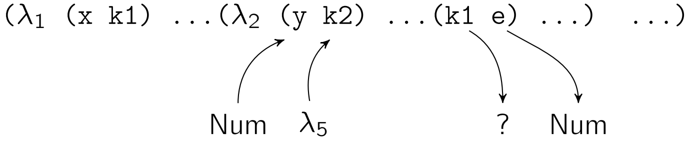

Callers: $(\lambda_2, \lambda_5, \text{Num})$, $(\lambda_1, \lambda_4, \text{Str})$, $(\lambda_1, \lambda_7, \text{Bool})$
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Thank you!